Q. 64 If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) $I + A$ (d) $3A$

Sol. *(a)* We have, $A^2 = I$

$$\therefore (A - I)^3 + (A + I)^3 - 7A = [(A - I) + (A + I)\{(A - I)^2 + (A + I)^2 - (A - I)(A + I)\}] - 7A$$

$$[\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$$

$$= [(2A)\{A^2 + I^2 - 2AI + A^2 + I^2 + AI - (A^2 - I^2)\}] - 7A$$

$$= 2A[I + I^2 + I + I^2 - A^2 + I^2] - 7A \qquad [\because A^2 = AI]$$

$$= 2A[5I - I] - 7A$$

$$= 8AI - 7AI \qquad [\because A = AI]$$

$$= AI = A$$

Q. 65 For any two matrices A and B, we have

(a) $AB = BA$ (b) $AB \neq BA$ (c) $AB = O$ (d) None of these

Sol. *(d)* For any two matrices A and B, we may have $AB = BA = I$, $AB \neq BA$ and $AB = O$ but it is not always true.

Q. 66 On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have

(a) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

Sol. *(d)* Given that, $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

On using $C_2 \to C_2 - 2C_1$, $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

Since, on using elementary column operation on $X = AB$, we apply these operations simultaneously on X and on the second matrix B of the product AB on RHS.

Q. 67 On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, we have

(a) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 2 \\ -5 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Sol. *(b)* Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$\therefore$

$$A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = A$$

So, the given matrix is a symmetric matrix.

[since, in a square matrix A, if $A' = A$, then A is called symmetric matrix]

Q. 61 The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

(a) diagonal matrix
(b) symmetric matrix
(c) skew-symmetric matrix
(d) scalar matrix

Sol. *(c)* We know that, in a square matrix, if $b_{ij} = 0$, when $i \neq j$, then it is said to be a diagonal matrix. Here, $b_{12}, b_{13}, \ldots \neq 0$, so the given matrix is not a diagonal matrix.

Now,

$$B = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$\therefore$

$$B' = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix} = -B$$

So, the given matrix is a skew-symmetric matrix, since we know that in a square matrix B, if $B' = -B$, then it is called skew-symmetric matrix.

Q. 62 If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is

(a) $m \times m$
(b) $n \times n$
(c) $n \times m$
(d) $m \times n$

Sol. *(d)* Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{p \times q}$

$\therefore$ $B' = [b_{ji}]_{q \times p}$

Now, AB' is defined, so $n = q$

and $B'A$ is also defined, so $p = m$

$\therefore$ Order of $B' = [b_{ji}]_{n \times m}$

and order of $B = [b_{ij}]_{m \times n}$

Q. 63 If A and B are matrices of same order, then $(AB' - BA')$ is a

(a) skew-symmetric matrix
(b) null matrix
(c) symmetric matrix
(d) unit matrix

Sol. *(a)* We have matrices A and B of same order.

Let $P = (AB' - BA')$

Then,

$$\begin{aligned} P' &= (AB' - BA')' = (AB')' - (BA')' \\ &= (B')'(A)' - (A')'B' = BA' - AB' \\ &= -(AB' - BA') = -P \end{aligned}$$

Hence, $(AB' - BA')$ is a skew-symmetric matrix.

$$\therefore \quad A-B=\begin{bmatrix} \frac{1}{\pi}(\sin^{-1} x\pi + \cos^{-1} x\,\pi) & \frac{1}{\pi}\left(\tan^{-1}\frac{x}{\pi} - \tan^{-1}\frac{x}{\pi}\right) \\ \frac{1}{\pi}\left(\sin^{-1}\frac{x}{\pi} - \sin^{-1}\frac{x}{\pi}\right) & \frac{1}{\pi}\cot^{-1}\pi x + \tan^{-1}\pi x \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\pi} \cdot \frac{\pi}{2} & 0 \\ 0 & \frac{1}{\pi} \cdot \frac{\pi}{2} \end{bmatrix} \qquad \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \text{ and } \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{2} I$$

Q. 57 If A and B are two matrices of the order $3 \times m$ and $3 \times n$, respectively and $m = n$, then order of matrix $(5A - 2B)$ is

(a) $m \times 3$ (b) 3×3

(c) $m \times n$ (d) $3 \times n$

Sol. *(d)* $A_{3\times m}$ and $B_{3 \times n}$ are two matrices. If $m = n$, then A and B have same orders as $3 \times n$ each, so the order of $(5A - 2B)$ should be same as $3 \times n$.

Q. 58 If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol. *(d)* $\because A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q. 59 If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = 1$, if $i \neq j = 0$ and if $i = j$, then A^2 is equal to

(a) I (b) A

(c) 0 (d) None of these

Sol. *(a)* We have, $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = 1$, if $i \neq j = 0$ and if $i = j$

$$\therefore \qquad A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and
$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Q. 60 The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a

(a) identity matrix (b) symmetric matrix

(c) skew-symmetric matrix (d) None of these

Objective Type Questions

Q. 53 The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

(a) square matrix (b) diagonal matrix

(c) unit matrix (d) None of these

Sol. *(a)* We know that, in a square matrix number of rows are equal to the number of columns, so the matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a square matrix.

Q. 54 Total number of possible matrices of order 3×3 with each entry 2 or 0 is

(a) 9 (b) 27

(c) 81 (d) 512

Sol. *(d)* Total number of possible matrices of order 3×3 with each entry 2 or 0 is 2^9 *i.e.*, 512.

Q. 55 $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of $x + y$ is

(a) $x = 3, y = 1$ (b) $x = 2, y = 3$

(c) $x = 2, y = 4$ (d) $x = 3, y = 3$

Sol. *(b)* We have, $4x = x + 6 \Rightarrow x = 2$

and $4x = 7y - 13 \Rightarrow 8 = 7y - 13$

$\Rightarrow$ $7y = 21 \Rightarrow y = 3$

$\therefore$ $x + y = 2 + 3 = 5$

Q. 56 If $A = \dfrac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\dfrac{x}{\pi}\right) \\ \sin^{-1}\left(\dfrac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ and $B = \dfrac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\dfrac{x}{\pi}\right) \\ \sin^{-1}\left(\dfrac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$,

then $A - B$ is equal to

(a) I (b) 0 (c) $2I$ (d) $\dfrac{1}{2}I$

Sol. *(d)* We have, $A = \begin{bmatrix} \dfrac{1}{\pi}\sin^{-1} x\pi & \dfrac{1}{\pi}\tan^{-1}\dfrac{x}{\pi} \\ \dfrac{1}{\pi}\sin^{-1}\dfrac{x}{\pi} & \dfrac{1}{\pi}\cot^{-1}\pi x \end{bmatrix}$

and $B = \begin{bmatrix} \dfrac{-1}{\pi}\cos^{-1} x\pi & \dfrac{1}{\pi}\tan^{-1}\dfrac{x}{\pi} \\ \dfrac{1}{\pi}\sin^{-1}\dfrac{x}{\pi} & \dfrac{-1}{\pi}\tan^{-1}\pi x \end{bmatrix}$

Q. 52 Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Thinking Process

We know that, any square matrix A can be expressed as the sum of a symmetric matrix and skew-symmetric matrix, i.e., $A = \dfrac{A + A'}{2} + \dfrac{A - A'}{2}$, *where* $A + A'$ *and* $A - A'$ *are a symmetric matrix and a skew-symmetric matrix, respectively.*

Sol. We have,
$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$$

$\therefore$
$$A' = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now,
$$\frac{A + A'}{2} = \frac{1}{2}\begin{bmatrix} 4 & 4 & 5 \\ 4 & -2 & 3 \\ 5 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix}$$

and
$$\frac{A - A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

$\therefore$
$$\frac{A + A'}{2} + \frac{A - A'}{2} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

which is the required expression.

(ii) $\therefore$

$$\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \begin{bmatrix} \because R_2 \to R_2 + R_3 \\ \text{and } R_1 \to R_1 - 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \qquad [\because R_2 \to R_2 + R_1]$$

Since, second row of the matrix A on LHS is containing all zeroes, so we can say that inverse of matrix A does not exist.

(iii) $\therefore$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad [\because R_2 \to R_2 - R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \qquad \begin{bmatrix} \because R_2 \to R_2 - R_1 \\ \text{and } R_3 \to R_3 + R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 4 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \qquad \begin{bmatrix} \because R_3 \to R_3 + R_1 \\ \text{and } R_2 \to R_2 - \frac{1}{2} R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad [\because R_3 \to R_3 - 2R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A \qquad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A \qquad \begin{bmatrix} \because R_1 \to \frac{1}{2} R_1 \\ \text{and} R_3 \to 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \qquad \begin{bmatrix} \because R_1 \to R_1 + \frac{1}{2} R_3 \\ \text{and } R_2 \to R_2 - \frac{5}{2} R_3 \end{bmatrix}$$

Hence, $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ is the inverse of given matrix A.

Q. 51 If possible, using elementary row transformations, find the inverse of the following matrices.

(i) $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Sol. For getting the inverse of the given matrix A by row elementary operations we may write the given matrix as

$$A = IA$$

(i) $\because \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_1 \to R_1 + R_2]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ 0 & -1 & -17 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -5 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 - 3R_1]$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & -10 \\ 0 & -1 & -17 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 0 \\ -5 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to R_1 + R_2 \\ \text{and } R_3 \to -1 \cdot R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ 1 & 1 & -1 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to R_1 + 10R_3 \\ \text{and } R_2 \to R_2 + 17R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to -1R_1 \\ \text{and } R_2 \to -1R_2 \end{bmatrix}$$

So, the inverse of A is $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$.

$\therefore$ $$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{3y} & \frac{1}{6y} & \frac{-1}{6y} \\ \frac{1}{3z} & \frac{-1}{3z} & \frac{1}{3z} \end{bmatrix} = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$\Rightarrow$ $$\frac{1}{2x} = x \Rightarrow \; = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\frac{1}{6y} = y \Rightarrow y = \pm\frac{1}{\sqrt{6}}$$

and $$\frac{1}{3z} = z \Rightarrow z = \pm\frac{1}{\sqrt{3}}$$

Alternate Method

We have,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \text{ and } A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Also, $$A' = A^{-1}$$

$\Rightarrow$ $$AA' = AA^{-1} \qquad [\because AA^{-1} = I]$$

$\Rightarrow$ $$AA' = I$$

$\Rightarrow$ $$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 4y^2 + z^2 & 2y^2 - z^2 & -2y^2 + z^2 \\ 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ -2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$2y^2 - z^2 = 0 \Rightarrow 2y^2 = z^2$$

$\Rightarrow$ $$4y^2 + z^2 = 1$$

$\Rightarrow$ $$2 \cdot z^2 + z^2 = 1$$

$$z = \pm\frac{1}{\sqrt{3}}$$

$\therefore$ $$y^2 = \frac{z^2}{2} \Rightarrow y = \pm\frac{1}{\sqrt{6}}$$

Also, $$x^2 + y^2 + z^2 = 1$$

$\Rightarrow$ $$x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3}$$

$$= 1 - \frac{3}{6} = \frac{1}{2}$$

$\Rightarrow$ $$x = \pm\frac{1}{\sqrt{2}}$$

$\therefore$ $$x = \pm, \frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$$

and $$z = \pm\frac{1}{\sqrt{3}}$$

Q. 50 Find x, y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

Sol. We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

By using elementary row transformations, we get

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ 0 & -2y & 2z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} A \qquad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & 3y & 0 \\ 0 & 0 & 3z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} A \qquad \begin{bmatrix} \because R_3 \to R_3 + R_1 \\ \text{and } R_2 \to R_2 + R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -x & -y & z \\ x & 3y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad \begin{bmatrix} \because R_1 \to R_1 - R_2 \\ \text{and } R_3 \to \frac{1}{3} R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -x & -y & 0 \\ x & 3y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} \frac{-1}{3} & \frac{-2}{3} & \frac{-1}{3} \\ 1 & 1 & 0 \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because R_1 \to R_1 - R_3]$$

$$\Rightarrow \begin{bmatrix} -x & -y & 0 \\ 0 & 2y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} \frac{-1}{3} & \frac{-2}{3} & \frac{-1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} -x & 0 & 0 \\ 0 & 2y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad \left[\because R_1 \to R_1 + \frac{1}{2} R_2\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{3y} & \frac{1}{6y} & \frac{-1}{6y} \\ \frac{1}{3z} & \frac{-1}{3z} & \frac{1}{3z} \end{bmatrix} A \qquad \begin{bmatrix} \because R_1 \to \frac{-1}{x} R_1, \\ R_2 \to \frac{1}{2y} R_2 \\ \text{and } R_3 \to \frac{1}{z} R_3 \end{bmatrix}$$

Also, $$P(y) \cdot P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$
$$= \begin{bmatrix} \cos y \cdot \cos x - \sin y \cdot \sin x & \cos y \cdot \sin x + \sin y \cdot \cos x \\ -\sin y \cdot \cos x - \sin x \cdot \cos y & -\sin y \cdot \sin x + \cos y \cdot \cos x \end{bmatrix}$$
$$= \begin{bmatrix} \cos (x+y) & \sin (x+y) \\ -\sin (x+y) & \cos (x+y) \end{bmatrix} \quad \text{...(iii)}$$

Thus, we see from the Eqs. (i), (ii) and (iii) that,

$$P(x) \cdot P(y) = P(x+y) = P(y) \cdot P(x)$$ **Hence proved.**

Q. 47 If A is square matrix such that $A^2 = A$, then show that $(I + A)^3 = 7A + I$.

Sol. Since, $A^2 = A$ and $(I+A) \cdot (I+A) = I^2 + IA + AI + A^2$
$$= I^2 + 2AI + A^2$$
$$= I + 2A + A = I + 3A$$
and $$(I + A) \cdot (I + A)(I + A) = (I + A)(I + 3A)$$
$$= I^2 + 3AI + AI + 3A^2$$
$$= I + 4AI + 3A$$
$$= I + 7A = 7A + I$$ **Hence proved.**

Q. 48 If A, B are square matrices of same order and B is a skew-symmetric matrix, then show that $A'BA$ is skew-symmetric.

Sol. Since, A and B are square matrices of same order and B is a skew-symmetric matrix *i.e.,* $B' = -B$.

Now, we have to prove that $A'BA$ is a skew-symmetric matrix.

$\therefore$ $$A'BA' = A'BA' = BA'A'$$ $[\because AB' = B'A']$
$$= A'B'A = A'-BA = -A'BA$$

Hence, $A'BA$ is a skew-symmetric matrix.

Long Answer Type Questions

Q. 49 If $AB = BA$ for any two square matrices, then prove by mathematical induction that $(AB)^n = A^n B^n$.

Sol. Let $$P(n) : (AB)^n = A^n B^n$$
$\therefore$ $$P(1) : (AB)^1 = A^1 B^1 \Rightarrow AB = AB$$

So, $P(1)$ is true.

Now, $$P(k) : (AB)^k = A^k B^k, \ k \in N$$

So, $P(K)$ is true, whenever $P(k+1)$ is true.

$\therefore$ $$P(K+1 : AB)^{k+1} = A^{k+1} B^{k+1} \quad \text{...(i)}$$
$\Rightarrow$ $$AB^k \cdot AB^1 \quad [\because AB = BA]$$
$\Rightarrow$ $$A^k B^k \cdot BA \Rightarrow A^k B^{k+1} A$$
$\Rightarrow$ $$A^k \cdot A \cdot B^{k+1} \Rightarrow A^{k+1} B^{k+1}$$
$\Rightarrow$ $$(A \cdot B)^{k+1} = A^{k+1} B^{k+1}$$

So, $P(k+1)$ is true for all $n \in N$, whenever $P(k)$ is true.

By mathematical induction $(AB) = A^n B^n$ is true for all $n \in N$.

Q. 45 If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c.

Thinking Process

We know that, a matrix A is skew-symmetric matrix, if $A' = -A$, so by using this we can get the values of a, b and c.

Sol. Let $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$

Since, A is skew-symmetric matrix.

$$\therefore \quad A' = -A$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & +1 \\ -c & -1 & 0 \end{bmatrix}$$

By equality of matrices, we get

$$a = -2,\ c = -3 \text{ and } b = -b \Rightarrow b = 0$$

$$\therefore \quad a = -2,\ b = 0 \text{ and } c = -3$$

Q. 46 If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that $P(x) \cdot P(y) = P(x + y)$ $= P(y) \cdot P(x)$.

Sol. We have,

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\therefore \quad P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

Now, $$P(x) \cdot P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cdot \cos y - \sin x \cdot \sin y & \cos x \cdot \sin y + \sin x \cdot \cos y \\ -\sin x \cdot \cos y - \cos x \cdot \sin y & -\sin x \cdot \sin y + \cos x \cdot \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos (x + y) & \sin (x + y) \\ -\sin (x + y) & \cos (x + y) \end{bmatrix} \quad \text{...(i)}$$

$$\begin{bmatrix} \because \cos (x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y \\ \text{and } \sin (x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y \end{bmatrix}$$

and $$P(x + y) = \begin{bmatrix} \cos (x + y) & \sin (x + y) \\ -\sin (x + y) & \cos (x + y) \end{bmatrix} \quad \text{...(ii)}$$

$$\therefore \quad \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a+0d & b+0\cdot e & c+0\cdot f \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a & b & c \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

By equality of matrices, we get

$$a = 1, b = -2, c = -5$$

and $\quad 2a - d = -1 \Rightarrow d = 2a + 1 = 3;$

$\Rightarrow \quad 2b - e = -8 \Rightarrow e = 2(-2) + 8 = 4$

$$2c - f = -10 \Rightarrow f = 2c + 10 = 0$$

$$\therefore \quad A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

Q. 43 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$, then find $A^2 + 2A + 7I$.

Sol. We have, $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \qquad [\because A^2 = A \cdot A]$$

$$= \begin{bmatrix} 1+8 & 2+2 \\ 4+4 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix}$$

$$\therefore \quad A^2 + 2A + 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$$

Q. 44 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A^{-1} = A'$, then find the value of α.

Sol. We have,

$$A = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} \text{ and } A' = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

Also, $\quad A^{-1} = A'$

$\Rightarrow \quad AA^{-1} = AA'$

$$\Rightarrow \quad I = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

By using equality of matrices, we get

$$\cos^2\alpha + \sin^2\alpha = 1$$

which is true for all real values of α.

$$\therefore \quad A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $\quad A^2 - 5A - 14\,I = 0$

$\Rightarrow \quad A \cdot A^2 - 5\,A \cdot A - 14\,A\,I = 0$

$\Rightarrow \quad A^3 - 5A^2 - 14A = 0 \qquad [\because A\,I = A]$

$\Rightarrow \quad A^3 = 5A^2 = 14A$

$$= 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \qquad \text{[using Eqs. (i) and (ii)]}$$

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Q. 41 Find the values of a, b, c and d, if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2\;d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} z.$$

Sol. We have,

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

$\Rightarrow \quad \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ c+d-1 & 3+2\,d \end{bmatrix}$

$\Rightarrow \quad 3a = a + 4 \Rightarrow a = 2;$

$3b = 6 + a + b$

$\Rightarrow \quad 3b - b = 8 \Rightarrow b = 4;$

$3d = 3 + 2d \Rightarrow d = 3$

and $\Rightarrow \quad 3c = c + d - 1$

$\Rightarrow \quad 2c = 3 - 1\; c = 1$

$\therefore \quad a = 2,\; b = 4,\; c = 1 \text{ and } d = 3$

Q. 42 Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$.

Sol. We have, $\quad \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3 \times 2} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}_{3 \times 3}$

From the given equation, it is clear that order of A should be 2×3.

Let $\quad A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

Q. 38 If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then find the values of x, y, z and w.

Sol. We have, $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

By equality of matrix, $x + y = 6$ and $xy = 8$

$\Rightarrow$ $x = 6 - y$ and $(6 - y) \cdot y = 8$

$\Rightarrow$ $y^2 - 6y + 8 = 0$

$\Rightarrow$ $y^2 - 4y - 2y + 8 = 0$

$\Rightarrow$ $(y-2)(y-4) = 0$

$\Rightarrow$ $y = 2$ or $y = 4$

$\therefore$ $x = 6 - 2 = 4$

or $x = 6 - 4 = 2$ $[\because x = 6 - y]$

Also, $z + 6 = 0$

$\Rightarrow$ $z = -6$ and $w = 4$

$\therefore$ $x = 2, y = 4$ or $x = 4, y = 2, z = -6$ and $w = 4$

Q. 39 If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, then find a matrix C such that $3A + 5B + 2C$ is a null matrix.

Sol. We have, $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$

Let $C = \begin{bmatrix} a & b \\ C & d \end{bmatrix}$

$\therefore$ $3A + 5B + 2C = 0$

$\Rightarrow$ $\begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 48+2a & 20+2b \\ 56+2c & 76+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow$ $2a + 48 = 0 \Rightarrow a = -24$

Also, $20 + 2b = 0 \Rightarrow b = -10$

$56 + 2c = 0 \Rightarrow c = -28$

and $76 + 2d = 0 \Rightarrow d = -38$

$\therefore$ $C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

Q. 40 If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

Sol. We have, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ ...(i)

$\therefore$ $A^2 = A \cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$ ...(ii) where $P(k+1)$ is true whenever $P(k)$ is true.

$$\therefore \quad P(k+1): (A')^k . (A')^1 = [A^{k+1}]'$$
$$(A^k)' . (A)' = [A^{k+1}]'$$
$$(A \cdot A^k)' = [A^{k+1}]' \qquad [\because (A')^k = (A^k)' \text{ and } (AB) = B'A']$$
$$(A^{k+1})' = [A^{k+1}]'$$

Hence proved.

Q. 37 Find inverse, by elementary row operations (if possible), of the following matrices.

(i) $\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

Thinking Process

To find the inverse of a matrix A, we know that $A = IA$ is used for elementary row operations. So, with the help of this method we can get the desired result.

Sol. (i) Let $A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$

In order to use elementary row operations we may write $A = IA$.

$$\therefore \quad \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 0 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} A \qquad [\because R_2 \to R_2 + 5R_1]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5/22 & 1/22 \end{bmatrix} A \qquad \left[\because R_2 \to \frac{1}{22} R_2\right]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7/22 & -3/22 \\ 5/22 & 1/22 \end{bmatrix} A \qquad [\because R_1 \to R_1 - 3R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix} A$$

$\Rightarrow$ $I = BA$, where B is the inverse of A.

$$\therefore \quad B = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

In order to use elementary row operations, we write $A = IA$

$$\Rightarrow \quad \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} A \qquad [\because R_2 \to R_2 + 2R_1]$$

Since, we obtain all zeroes in a row of the matrix A on LHS, so A^{-1} does not exist.

Q. 34 If $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $x^2=-1$, then show that $(A+B)^2=A^2+B^2$.

Sol. We have, $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $x^2=-1$

$\therefore$ $(A+B)=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}$

and $(A+B)^2=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}$

$=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix}$ ...(i)

Also, $A^2=A\cdot A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}=\begin{bmatrix}-x^2 & 0\\ 0 & -x^2\end{bmatrix}$

and $B^2=B\cdot B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Now, $A^2+B^2=\begin{bmatrix}-x^2+1 & 0\\ 0 & -x^2+1\end{bmatrix}=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix}$ [using Eq. (i)]

$=(A+B)^2$ **Hence proved.**

Q. 35 Verify that $A^2=I$, when $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$.

Sol. We have, $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$

$\therefore$ $A^2=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}\cdot\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$ [$\because A^2=A\cdot A$]

$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$ **Hence proved.**

Q. 36 Prove by mathematical induction that $(A')^n=(A^n)'$ where $n\in N$ for any square matrix A.

Sol. Let $P(n):\ (A')^n=(A^n)'$

$\therefore$ $P(1):\ (A')^1=(A)'$

$\Rightarrow$ $A'=A'\Rightarrow P(1)$ is true.

Now, $P(k):(A')^k=(A^k)'$,

where $k\in N$

and $P(k+1):(A')^{k+1}=(A^{k+1})'$

(vii) $AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$

$\therefore \quad (AB)^T = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$

Now, $\quad B^T A^T = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$

$= (AB)^T$ **Hence proved.**

(viii) $(A-B) = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}$

$(A-B)\ C = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$ ...(i)

Now, $\quad AC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix}$ ...(ii)

and $\quad BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$ ...(iii)

$\therefore \quad AC - BC = \begin{bmatrix} 4-8 & -4-0 \\ 1-7 & -6+10 \end{bmatrix}$ [using Eqs. (ii) and (iii)]

$= \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$

$= (A-B)C$ [using Eq. (i)] **Hence proved.**

(ix) $(A-B)^T = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix}^T$

$= \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix}$

$A^T - B^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix} = (A-B)^T$ **Hence proved.**

Q. 33 If $A = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$, then show that $A^2 = \begin{bmatrix} \cos 2q & \sin 2q \\ -\sin 2q & \cos 2q \end{bmatrix}$.

Sol. We have, $A = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix} \cdot \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$

$= \begin{bmatrix} \cos^2 q - \sin^2 q & \cos q \cdot \sin q + \sin q \cos q \\ -\sin q \cos q - \cos q \sin q & -\sin^2 q + \cos^2 q \end{bmatrix}$

$= \begin{bmatrix} \cos 2q & 2\sin q \cos q \\ -2\sin q \cos q & \cos 2q \end{bmatrix}$ $[\because \cos^2\theta - \sin^2\theta = \cos 2\theta]$

$= \begin{bmatrix} \cos 2q & \sin 2q \\ -\sin 2q & \cos 2q \end{bmatrix}$ $[\because \sin 2\theta = 2\sin\theta \cdot \cos\theta]$ **Hence proved.**

(ii) $(BC)=\begin{bmatrix}4&0\\1&5\end{bmatrix}\begin{bmatrix}2&0\\1&-2\end{bmatrix}=\begin{bmatrix}8&0\\7&-10\end{bmatrix}$

and $A\ (BC)=\begin{bmatrix}1&2\\-1&3\end{bmatrix}\begin{bmatrix}8&0\\7&-10\end{bmatrix}$

$=\begin{bmatrix}8+14&0-20\\-8+21&0-30\end{bmatrix}=\begin{bmatrix}22&-20\\13&-30\end{bmatrix}$

Also, $(AB)=\begin{bmatrix}1&2\\-1&3\end{bmatrix}\cdot\begin{bmatrix}4&0\\1&5\end{bmatrix}=\begin{bmatrix}6&10\\-1&15\end{bmatrix}$

$(AB)\ C=\begin{bmatrix}6&10\\-1&15\end{bmatrix}\begin{bmatrix}2&0\\1&-2\end{bmatrix}$

$=\begin{bmatrix}22&-20\\13&-30\end{bmatrix}=A(BC)$ **Hence proved.**

(iii) $(a+b)\ B=(4-2)\begin{bmatrix}4&0\\1&5\end{bmatrix}$ $[\because a=4,\ b=-2]$

$=\begin{bmatrix}8&0\\2&10\end{bmatrix}$

and $aB+bB=4B-2B$

$=\begin{bmatrix}16&0\\4&20\end{bmatrix}-\begin{bmatrix}8&0\\2&10\end{bmatrix}$

$=\begin{bmatrix}8&0\\2&10\end{bmatrix}$

$=(a+b)B$ **Hence proved.**

(iv) $(C-A)=\begin{bmatrix}2-1&0-2\\1+1&-2-3\end{bmatrix}=\begin{bmatrix}1&-2\\2&-5\end{bmatrix}$

and $a(C-A)=\begin{bmatrix}4&-8\\8&-20\end{bmatrix}$ $[\because a=4]$

Also, $aC-aA=\begin{bmatrix}8&0\\4&-8\end{bmatrix}-\begin{bmatrix}4&8\\-4&12\end{bmatrix}=\begin{bmatrix}4&-8\\8&-20\end{bmatrix}$

$=a(C-A)$ **Hence proved.**

(v) $A^T=\begin{bmatrix}1&2\\-1&3\end{bmatrix}^T=\begin{bmatrix}1&-1\\2&3\end{bmatrix}$

Now, $(A^T)^T=\begin{bmatrix}1&2\\-1&3\end{bmatrix}^T$

$=A$ **Hence proved.**

(vi) $(bA)^T=\begin{bmatrix}-2&-4\\2&-6\end{bmatrix}^T$ $[\because\ b=-2]$

$=\begin{bmatrix}-2&2\\-4&-6\end{bmatrix}$

and $A^T=\begin{bmatrix}1&-1\\2&3\end{bmatrix}$

$\therefore$ $bA^T=\begin{bmatrix}-2&2\\-4&-6\end{bmatrix}=(bA)^T$ **Hence proved.**

Q. 30 Let A and B be square matrices of the order 3×3. Is $(AB)^2 = A^2B^2$? Give reasons.

Sol. Since, A and B are square matrices of order 3×3.

$$\therefore \quad AB^2 = AB \cdot AB$$
$$= ABAB$$
$$= AABB \quad [\because AB = BA]$$
$$= A^2B^2$$

So, $AB^2 = A^2B^2$ is true when $AB = BA$.

Q. 31 Show that, if A and B are square matrices such that $AB = BA$, then $(A+B)^2 = A^2 + 2AB + B^2$.

Sol. Since, A and B are square matrices such that $AB = BA$.

$$\therefore \quad (A+B)^2 = (A+B)\cdot(A+B)$$
$$= A^2 + AB + BA + B^2$$
$$= A^2 + AB + AB + B^2 \quad [\because AB = BA]$$
$$= A^2 + 2AB + B^2$$

Hence proved.

Q. 32 If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}, a = 4,$ and $b = -2$, then show that

(i) $A + (B + C) = (A + B) + C$

(ii) $A(BC) = (AB)C$

(iii) $(a + b)B = aB + bB$

(iv) $a(C - A) = aC - aA$

(v) $(A^T)^T = A$

(vi) $(bA)^T = bA^T$

(vii) $(AB)^T = B^T A^T$

(viii) $(A - B)C = AC - BC$

(ix) $(A - B)^T = A^T - B^T$

Sol. We have,
$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} \text{ and } a = 4, b = -2$$

(i) $A + (B + C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

and
$$(A + B) + C = \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix} = A + (B + C)$$

Hence proved.

Q. 28 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then verify that

(i) $(2A + B)' = 2AA + B'$.

(ii) $(A - B)' = A' - B'$.

Sol. We have, $$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$$

(i) $\therefore$ $$(2A + B) = \begin{bmatrix} 2 & 4 \\ 8 & 2 \\ 10 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 14 & 6 \\ 17 & 15 \end{bmatrix}$$

and $$(2A + B)' = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$$

Also, $$2A' + B' = 2\begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix} = (2A + B)'$$ **Hence proved.**

(ii) $$(A - B) = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & -3 \\ -2 & 3 \end{bmatrix}$$

and $$(A - B)' = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$$

Also, $$A' - B' = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$$

$$= (A - B)'$$ **Hence proved.**

Q. 29 Show that $A' A$ and $A A'$ are both symmetric matrices for any matrix A.

Thinking Process

We know that, for a matrix A to be symmetric matrix, $A' = A$. Also by using the result $(AB)' = BA'$, we can prove that $A'A$ and AA' are both symmetric matrices for any matrix A.

Sol. Let $$P = A'A$$

$\therefore$ $$P' = (AA')'$$
$$= A'(A')' \quad [\because (AB')' = B'A']$$
$$= A'A = P$$

So, $A'A$ is symmetric matrix for any matrix A.

Similarly, let $$Q = A A'$$

$\therefore$ $$Q' = (AA')' = (A')'(A)'$$
$$= A(A')' = Q$$

So, AA' is symmetric matrix for any matrix A.

Q. 27 If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that

(i) $(A')' = A$
(ii) $(AB)' = B'A'$
(iii) $(kA)' = (kA')$.

Sol. We have, $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$

(i) We have to verify that, $A' = A$

$$\therefore \quad A' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$$

and
$$A' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} = A$$

Hence proved.

(ii) We have to verify that, $AB' = B'A'$

$$\therefore \quad AB = \begin{bmatrix} 3 & 9 \\ 11 & -15 \end{bmatrix}$$

$$\Rightarrow \quad (AB)' = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$$

and
$$B'A' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$$

$$= (AB)'$$

Hence proved.

(iii) We have to verify that, $(kA)' = (kA')$

Now,
$$(kA) = \begin{bmatrix} 0 & -k & 2k \\ 4k & 3k & -4k \end{bmatrix}$$

and
$$(kA)' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$$

Also,
$$kA' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$$

$$= (kA)'$$

Hence proved.

$\therefore \quad A(B+C)=[2 \quad 1]\begin{bmatrix}5-1 & 3+2 & 4+1\\ 8+1 & 7+0 & 6+2\end{bmatrix}$

$=[2\ 1]\begin{bmatrix}4 & 5 & 5\\ 9 & 7 & 8\end{bmatrix}$

$=[8+9 \quad 10+7 \quad 10+8]$

$=[17\ 17\ 18]$ ...(i)

Also, $AB=[2\ 1]\begin{bmatrix}5 & 3 & 4\\ 8 & 7 & 6\end{bmatrix}$

$=[10+8 \quad 6+7 \quad 8+6]=[18 \quad 13 \quad 14]$

and $AC=[2\ 1]\begin{bmatrix}-1 & 2 & 1\\ 1 & 0 & 2\end{bmatrix}$

$=[-2+1 \quad 4+0 \quad 2+2]=[-1 \quad 4 \quad 4]$

$\therefore \quad AB+AC=[18 \quad 13 \quad 14]+[-1 \quad 4 \quad 4]$

$=[17 \quad 17 \quad 18]$ ...(ii)

$\therefore \quad A(B+C)=(AB+AC)$ [using Eqs. (i) and (ii)]

Hence proved.

Q. 26 If $A=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$, then verify that $A^2+A=(A+I)$, where I is 3×3 unit matrix.

Sol. We have, $A=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$

$\therefore \quad A^2=A\cdot A$

$=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & -1 & -2\\ 4 & 4 & 4\\ 2 & 2 & 4\end{bmatrix}$

$\therefore \quad A^2+A=\begin{bmatrix}1 & -1 & -2\\ 4 & 4 & 4\\ 2 & 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$

$=\begin{bmatrix}2 & -1 & -3\\ 6 & 5 & 7\\ 2 & 3 & 5\end{bmatrix}$ ...(i)

Now, $A+I=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}2 & 0 & -1\\ 2 & 2 & 3\\ 0 & 1 & 2\end{bmatrix}$

and $A(A+I)=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}\cdot\begin{bmatrix}2 & 0 & -1\\ 2 & 2 & 3\\ 0 & 1 & 2\end{bmatrix}=\begin{bmatrix}2 & -1 & -3\\ 6 & 5 & 7\\ 2 & 3 & 5\end{bmatrix}$ ...(ii)

Thus, we see that $A^2+A=A(A+I)$ [using Eqs. (i) and (ii)]

Q. 23 If $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then prove that

$$PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Sol. $PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$ ...(i)

and $QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$ ...(ii)

Thus, we see that, $PQ = QP$ [using Eqs. (i) and (ii)]

Hence proved.

Q. 24 If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then find the value of A.

Sol. We have, $[2\ 1\ 3] \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$

$\therefore$ $[2\,1\,3] \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = [-2-1+0 \quad 0+1+3 \quad -2+0+3]$

$= [-3 \quad 4 \quad 1]$

Now, $[-3\,4\,1] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$

$\therefore$ $A = [-3\,4\,1] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= [-3+0-1] = [-4]$

Q. 25 If $A = [2\ 1]$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then verify that

$A(B + C) = (AB + AC)$.

Sol. We have to verify that, $A(B + C) = AB + AC$

We have, $A = [2 \quad 1]$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

Q. 22 If $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, verify

(i) $(AB)\,C = A(BC)$.

(ii) $A\,(B + C) = AB + AC$.

Sol. We have, $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

(i) $(AB) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$

and $(AB)C = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 8+5 & 0 \\ -1+10 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix}$ ...(i)

Again, $(BC) = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2-3 & 0 \\ 3+4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$

and $A\,(BC) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1+14 & 0 \\ +2+7 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix}$ ...(ii)

$\therefore$ $(AB)\,C = A(BC)$ [using Eqs. (i) and (ii)]

(ii) $(B + C) = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$

and $A \cdot (B + C) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 3+4 & 3-8 \\ -6+2 & -6-4 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix}$ ...(iii)

Also, $AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$

and $AC = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1-2 & 0 \\ -2-1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$

$\therefore$ $AB + AC = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$

$\Rightarrow$ $AB + AC = \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix}$ ...(iv)

From Eqs. (iii) and (iv),

$A\,(B + C) = AB + AC$

Q. 20 If $A = [3\ 5]$ and $B = [7\ 3]$, then find a non-zero matrix C such that $AC = BC$.

Sol. We have, $A = [3\ \ 5]_{1\times 2}$ and $B = [7\ \ 3]_{1\times 2}$

Let $C = \begin{bmatrix} x \\ y \end{bmatrix}_{2\times 1}$ is a non-zero matrix of order 2×1.

$$\therefore \quad AC = [3\ \ 5]\begin{bmatrix} x \\ y \end{bmatrix} = [3x + 5y]$$

$$\text{and} \quad BC = [7\ \ 3]\begin{bmatrix} x \\ y \end{bmatrix} = [7x + 3y]$$

For $AC = BC$,

$$[3x + 5y] = [7x + 3y]$$

On using equality of matrix, we get

$$3x + 5y = 7x + 3y$$

$$\Rightarrow \quad 4x = 2y$$

$$\Rightarrow \quad x = \frac{1}{2}y$$

$$\Rightarrow \quad y = 2x$$

$$\therefore \quad C = \begin{bmatrix} x \\ 2x \end{bmatrix}$$

We see that on taking C of order $2 \times 1, 2 \times 2, 2 \times 3, \ldots$, we get

$$C = \begin{bmatrix} x \\ 2x \end{bmatrix}, \begin{bmatrix} x & x \\ 2x & 2x \end{bmatrix}, \begin{bmatrix} x & x & x \\ 2x & 2x & 2x \end{bmatrix} \ldots$$

In general,

$$C = \begin{bmatrix} k \\ 2k \end{bmatrix}, \begin{bmatrix} k & k \\ 2k & 2k \end{bmatrix} \text{etc...}$$

where, k is any real number.

Q. 21 Give an example of matrices A, B and C, such that $AB = AC$, where A is non-zero matrix but $B \neq C$.

Sol. Let $$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{and } C = \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} \quad [\because B \neq C]$$

$$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \quad \ldots\text{(i)}$$

$$\text{and} \quad AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \quad \ldots\text{(ii)}$$

Thus, we see that $AB = AC$ [using Eqs. (i) and (ii)]

where, A is non-zero matrix but $B \neq C$.

On subtracting Eq. (i) from Eq. (ii), we get

$$14y = 28 \Rightarrow y = 2$$

$\therefore$ $$2x + 3 \times 2 - 8 = 0$$

$\Rightarrow$ $$2x = 2 \Rightarrow \quad x = 1$$

$\therefore$ $$x = 1 \text{ and } y = 2$$

Q. 19 If X and Y are 2×2 matrices, then solve the following matrix equations for X and Y

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, \; 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}.$$

Sol. We have,

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad \text{...(i)}$$

and $$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$\therefore$ $$(3X + 2Y) - (2X + 3Y) = \begin{bmatrix} -2-2 & 2-3 \\ 1-4 & -5-0 \end{bmatrix}$$

$$(X - Y) = \begin{bmatrix} -4 & -1 \\ -3 & -5 \end{bmatrix} \quad \text{...(iii)}$$

On adding Eqs. (i) and (ii), we get

$$(5X + 5Y) = \begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix}$$

$\Rightarrow$ $$(X + Y) = \frac{1}{5}\begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{...(iv)}$$

On adding Eqs. (iii) and (iv), we get

$$(X - Y) + (X + Y) = \begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$$

$\Rightarrow$ $$2X = 2\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

$\therefore$ $$X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

From Eq. (iv),

$$\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix} + Y = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$\therefore$ $$Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \text{ and } X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

and $$BA = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3}$$

$$= \begin{bmatrix} 4 \times 2 + 1 & 4+2 & 8+4 \\ 4+3 & 2+6 & 4+12 \\ 2+2 & 1+4 & 2+8 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}$$

Q. 16 Show by an example that for $A \neq 0$, $B \neq 0$ and $AB = 0$.

Sol. Let $$A = \begin{bmatrix} 0 & -4 \\ 0 & 2 \end{bmatrix} \neq 0 \text{ and } B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} \neq 0$$

$\therefore$ $$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence proved.

Q. 17 Given, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$. is $(AB)' = B' A'$?

Sol. We have, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}_{3\times2}$

$\therefore$ $$AB = \begin{bmatrix} 2+8+0 & 8+32+0 \\ 3+18+6 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$$

and $$(AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad \text{...(i)}$$

Also, $$B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}_{2\times3} \text{ and } A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}_{3\times2}$$

$\therefore$ $$B'A' = \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad \text{...(ii)}$$

Thus, we see that, $(AB)' = B'A'$ [using Eqs. (i) and (ii)]

Q. 18 Solve for x and y, $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$.

Sol. We have, $$x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3 \cdot y \\ 5 \cdot y \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x & 3y & -8 \\ x & 5y & -11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ $$2x + 3y - 8 = 0$$

$$\Rightarrow 4x + 6y = 16 \quad \text{...(i)}$$

and $$x + 5y - 11 = 0$$

$$\Rightarrow 4x + 20y = 44 \quad \text{...(ii)}$$

$$\Rightarrow \quad 4x = -4 \Rightarrow x = -1,\ 4y = 8$$
$$\Rightarrow \quad y = 2 \text{ and } 4z = 4$$
$$\Rightarrow \quad z = 1$$
$$\therefore \quad A = [-1\ 2\ 1]$$

Q. 14 If $A \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify $(BA)^2 \neq B^2 A^2$.

Sol. We have, $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3\times 2}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$

$$\therefore \quad BA = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3} \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 6+1+4 & -8+1+0 \\ 3+2+8 & -4+2+0 \end{bmatrix} = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

and $$(BA)\cdot(BA) = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix} \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

$$\Rightarrow \quad (BA)^2 = \begin{bmatrix} 121-91 & -77+14 \\ 143-26 & -91+4 \end{bmatrix} = \begin{bmatrix} 30 & -63 \\ 117 & -87 \end{bmatrix} \qquad \text{...(i)}$$

Also, $$B^2 = B \cdot B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$$

So, B^2 is not possible, since the B is not a square matrix.
Hence, $(BA)^2 \neq B^2 A^2$.

Q. 15 If possible, find the value of BA and AB, where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}.$$

Sol. We have, $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times 2}$

So, AB and BA both are possible.
[since, in both $A \cdot B$ and $B \cdot A$, the number of columns of first is equal to the number of rows of second.]

$$\therefore \quad AB = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3} \cdot \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 8+2+2 & 2+3+4 \\ 4+4+4 & 1+6+8 \end{bmatrix} = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}$$

$$\Rightarrow \quad 4a + 2c - 6b - 3d = 0 \quad \ldots(ii)$$
$$\Rightarrow \quad -9a - 6c + 15b + 10d = 0 \quad \ldots(iii)$$
$$\Rightarrow \quad 6a + 4c - 9b - 6d = 1 \quad \ldots(iv)$$

On adding Eqs. (i) and (iv), we get

$$c + b - d = 2 \Rightarrow d = c + b - 2 \quad \ldots(v)$$

On adding Eqs. (ii) and (iii), we get

$$-5a - 4c + 9b + 7d = 0 \quad \ldots(vi)$$

On adding Eqs. (vi) and (iv), we get

$$a + 0 + 0 + d = 1 \Rightarrow d = 1 - a \quad \ldots(vii)$$

From Eqs. (v) and (vii),

$$c + b - 2 = 1 - a \Rightarrow a + b + c = 3 \quad \ldots(viii)$$
$$\Rightarrow \quad a = 3 - b - c$$

Now, using the values of a and d in Eq. (iii), we get

$$-9(3 - b - c) - 6c + 15b + 10(-2 + b + c) = 0$$
$$\Rightarrow \quad -27 + 9b + 9c - 6c + 15b - 20 + 10b + 10c = 0$$
$$\Rightarrow \quad 34b + 13c = 47 \quad \ldots(ix)$$

Now, using the values of a and d in Eq. (ii), we get

$$4(3 - b - c) + 2c - 6b - 3(b + c - 2) = 0$$
$$\Rightarrow \quad 12 - 4b - 4c + 2c - 6b - 3b - 3c + 6 = 0$$
$$\Rightarrow \quad -13b - 5c = -18 \quad \ldots(x)$$

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get

$$\begin{array}{c} -169b - 65c = -234 \\ \underline{170\,b + 65c = 235} \\ b = 1 \end{array}$$

$$\Rightarrow \quad -13 \times 1 - 5c = -18 \quad \text{[from Eq. (x)]}$$
$$\Rightarrow \quad -5c = -18 + 13 = -5 \Rightarrow c = 1$$
$$\therefore \quad a = 3 - 1 - 1 = 1 \text{ and } d = 1 - 1 = 0$$
$$\therefore \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Q. 13 Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

Sol. We have, $$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times1} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3}$$

Let $$A = [x\ y\ z]$$

$$\therefore \quad \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times1} [x\ y\ z]_{1\times3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3}$$

$$\Rightarrow \quad \begin{bmatrix} 4x & 4y & 4z \\ x & y & z \\ 3x & 3y & 3z \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

and $$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$\therefore$ $$A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3-0 & 1+6-7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$ **Hence proved.**

Since, $$A^2 - 3A - 7I = 0$$

$\Rightarrow$ $$A^{-1}[(A^2) - 3A - 7I] = A^{-1}0$$

$\Rightarrow$ $$A^{-1}A \cdot A - 3A^{-1}A - 7A^{-1}I = 0 \qquad [\because A^{-1}0 = 0]$$

$\Rightarrow$ $$IA - 3I - 7A^{-1} = 0 \qquad [\because A^{-1}A = I]$$

$\Rightarrow$ $$A - 3I - 7A^{-1} = 0 \qquad [\because A^{-1}I = A^{-1}]$$

$\Rightarrow$ $$-7A^{-1} = -A + 3I$$

$$= \begin{bmatrix} -5 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$\therefore$ $$A^{-1} = \frac{-1}{7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

Q. 12 Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thinking Process

We know that, if two matrices A and B of order $m \times n$ and $p \times q$ respectively are multiplied, then necessity condition to multiplication of $A \cdot B$ is $n = p$. So, by taking a matrix of correct order we can get the desired elements of the required matrix.

Sol. We have, $$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times 2} A \cdot \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

Let $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

$\therefore$ $$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} -6a-3c+10b+5d & 4a+2c-6b-3d \\ -9a-6c+15b+10d & 6a+4c-9b-6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$-6a - 3c + 10b + 5d = 1 \qquad \ldots(i)$$

Also, $$A^2 = A\cdot A$$
$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\cdot\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

and $B^2 = B\cdot B$ $$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\therefore$ $$A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \text{...(ii)}$$

Thus, we see that
$$(A+B)\cdot(A-B) \neq A^2 - B^2 \quad \text{[using Eqs. (i) and (ii)]}$$
$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Hence proved.

Q. 10 Find the value of x, if $[1\ x\ 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$.

Sol. We have, $$[1\ x\ 1]_{1\times 3}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}_{3\times 3}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow [1+2x+15 \quad 3+5x+3 \quad 2+x+2]_{1\times 3}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow [16+2x \quad 5x+6 \quad x+4]_{1\times 3}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow [16+2x+(5x+6)\cdot 2+(x+4)\cdot x]_{1\times 1} = 0$$
$$\Rightarrow [16+2x+10x+12+x^2+4x] = 0$$
$$\Rightarrow [x^2+16x+28] = 0$$
$$\Rightarrow [x^2+2x+14x+28] = 0$$
$$\Rightarrow (x+2)(x+14) = 0$$
$$\therefore x = -2, -14$$

Q. 11 Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = O$ and hence find the value of A^{-1}.

Sol. We have, $$A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$\therefore$ $$A^2 = A\cdot A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\cdot\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

(ii) $\because\ 2X = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix}$

and $$3Y = 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$\therefore$ $$2X - 3Y = \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(iii) $X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & +1 \end{bmatrix}$

Also, $$X + Y + Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We see that Z is the additive inverse of $(X+Y)$ or negative of $(X+Y)$.

$\therefore$ $$Z = \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix} \qquad [\because Z = -(X+Y)]$$

Q. 8 Find non-zero values of x satisfying the matrix equation

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ (10) & 6x \end{bmatrix}.$$

Sol. Given that,

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ 10 & 6x \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 2x^2+16 & 2x+10x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$\Rightarrow$ $$2x + 10x = 48$$

$\Rightarrow$ $$12x = 48$$

$\therefore$ $$x = \frac{48}{12} = 4$$

Q. 9 If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then show that

$$(A+B)\ (A-B) \neq A^2 - B^2.$$

Sol. We have, $$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\therefore$ $$(A+B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}_{2\times 2}$$

and $$(A-B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

Since, $(A+B)\cdot(A-B)$ is defined, if the number of columns of $(A+B)$ is equal to the number of rows of $(A-B)$, so here multiplication of matrices $(A+B)\cdot(A-B)$ is possible.

Now, $$(A+B)_{2\times 2}\cdot(A-B)_{2\times 2} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \qquad \ldots(i)$$

3

Matrices

Short Answer Type Questions

Q. 1 If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Sol. We know that, if a matrix is of order $m \times n$, it has mn elements, where m and n are natural numbers.

We have, $m \times n = 28$

$$\Rightarrow \quad (m, n) = \{(1, 28), (2, 14), (4, 7), (7, 4), (14, 2), (28, 1)\}$$

So, the possible orders are $1 \times 28, 2 \times 14, 4 \times 7, 7 \times 4, 14 \times 2, 28 \times 1$.

Also, if it has 13 elements, then $m \times n = 13$

$$\Rightarrow \quad (m, n) = \{(1,13), (13,1)\}$$

Hence, the possible orders are $1 \times 13, 13 \times 1$.

Q. 2 In the matrix $A = \begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \dfrac{-2}{5} \end{bmatrix}$, write

(i) the order of the matrix A.

(ii) the number of elements.

(iii) elements a_{23}, a_{31} and a_{12}.

Sol. We have, $$A = \begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \dfrac{-2}{5} \end{bmatrix}$$

(i) the order of matrix $A = 3 \times 3$

(ii) the number of elements $= 3 \times 3 = 9$

[since, the number of elements in an $m \times n$ matrix will be equal to $m \times n = mn$]

(iii) $a_{23} = x^2 - y$, $a_{31} = 0$, $a_{12} = 1$

[since, we know that a_{ij}, is a representation of element lying in the ith row and jth column]